EXERCISE 3
OPEN PROBLEMS IN NUMBER THEORY 2017/18
DUE DATE: APRIL 26, 2018

Exercise 1. *Show that*

$$\sum_{\substack{m \in \mathbb{Z}^2 \\ 0 < |m| < M}} \frac{1}{|m|^{3/2}} \ll M^{1/2}.$$

Exercise 2. *The Bessel function* $J_0(z) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{z}{2}\right)^{2m}$ *admits an integral representation*

$$J_0(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-iz \sin t} dt.$$

Show that as $z \to +\infty$, $J_0(z) \ll 1/\sqrt{z}$.

Exercise 3. *Show that in dimension 3, the Fourier transform of the unit ball satisfies*

$$\int_{|x| \leq 1} e^{i\langle x, \xi \rangle} d^3 x \ll \frac{1}{|\xi|^2}, \quad \xi \in \mathbb{R}^3, |\xi| \geq 1.$$

Hint: You can directly evaluate the Fourier transform here!